

Monte Carlo Simulation of Neoclassical Confinement in Finite- β Helias Equilibria

W. Lotz, J. Nührenberg, and A. Schlüter

Max-Planck-Institut für Plasmaphysik, IPP-EURATOM Association,
D-8046 Garching bei München

Z. Naturforsch., **42 a**, 1045–1050 (1987); received June 15, 1987

Dedicated to Professor Dieter Pfirsch on his 60th Birthday

The beneficial effect of finite β on neoclassical confinement is demonstrated in Helias configurations. Equilibrium and transport calculations were performed for $\langle\beta\rangle$ up to 0.15.

1. Introduction

Monte Carlo simulation of neoclassical confinement developed previously [1] is a useful tool for demonstrating the influence of the structure of the confining magnetic field on confinement properties in the long-mean-free-path (lmfp) regime. Figure 1 shows three very different types of confinement behaviour demonstrated in an axisymmetric tokamak and in a tokamak with ripple: banana transport, v^{-1} transport owing to the ripple in an intermediate lmfp regime where real space diffusion prevails, and v transport in the very lmfp regime which is governed by velocity space diffusion. The latter two types of transport behaviour are also characteristic of toroidal stellarators, as is shown in Figure 2. The occurrence of a loss rate S approximately given by the collision frequency ν (ν particle velocity, Λ mean free path)

$$S_{\nu} \approx \nu/\Lambda = \nu$$

is of particular importance because it would represent a severe obstacle to fusion in stellarators. According to the current status of transport theory and simulation, an electric field in the form of an electric potential which is constant on magnetic surfaces and is of the order of the particle energy is the main healing mechanism that improves the transport behaviour in the lmfp regime. Figure 3 demonstrates an improvement of about two orders of magnitude in an $l=2$ stellarator in which a potential difference of twice the particle energy acts

on ions with a plasma- to gyro-radius ratio Q_{ρ} of $Q_{\rho} = 500$. An improvement of this order is required to allow self-sustained thermonuclear fusion.

The search for finite β effects on neoclassical confinement hitherto led to the conclusion that they are not important [2] or that they may even destroy the good transport properties of transport-optimized vacuum field configurations [3]. Analyzing Helias configurations, we show that these equilibria exhibit improved transport with increasing β in all transport regimes.

2. Finite- β Helias Equilibria

The properties of the Helias [4–7] stellarator can be summarized as follows: A significant magnetic well ($\gtrsim 1\%$) in the vacuum field, in combination with a strong reduction of the parallel current density ($\langle j_{\parallel}^2/j_{\perp}^2 \rangle \lesssim 0.5$), allows finite- β configurations with $\langle\beta\rangle \approx 0.1$ at acceptable aspect ratio ($\lesssim 12$) and otherwise moderate shape characterized by half-axis ratios not exceeding $\lesssim 3$ and moderate indentation. At these beta values the Shafranov shift and the change in shear are negligible, the change in twist is $\lesssim \langle\beta\rangle$, and the resistive interchange stability criterion as the most stringent necessary criterion and as conjectured measure of complete MHD stability is satisfied. Both options, finite shear with the occurrence of rational twist values of medium order (e.g. $\iota = 1/6$) and small shear avoiding such twist values, are available. Furthermore, the number of particles localized in the regions of weak curvature of the plasma column differs for different members of the Helias class. The Helias class of

Reprint requests to Dr. W. Lotz, Max-Planck-Institut für Plasmaphysik, D-8046 Garching bei München.

0932-0784 / 87 / 1000-1045 \$ 01.30/0. – Please order a reprint rather than making your own copy.



Dieses Werk wurde im Jahr 2013 vom Verlag Zeitschrift für Naturforschung in Zusammenarbeit mit der Max-Planck-Gesellschaft zur Förderung der Wissenschaften e.V. digitalisiert und unter folgender Lizenz veröffentlicht: Creative Commons Namensnennung-Keine Bearbeitung 3.0 Deutschland Lizenz.

Zum 01.01.2015 ist eine Anpassung der Lizenzbedingungen (Entfall der Creative Commons Lizenzbedingung „Keine Bearbeitung“) beabsichtigt, um eine Nachnutzung auch im Rahmen zukünftiger wissenschaftlicher Nutzungsformen zu ermöglichen.

This work has been digitalized and published in 2013 by Verlag Zeitschrift für Naturforschung in cooperation with the Max Planck Society for the Advancement of Science under a Creative Commons Attribution-NoDerivs 3.0 Germany License.

On 01.01.2015 it is planned to change the License Conditions (the removal of the Creative Commons License condition “no derivative works”). This is to allow reuse in the area of future scientific usage.

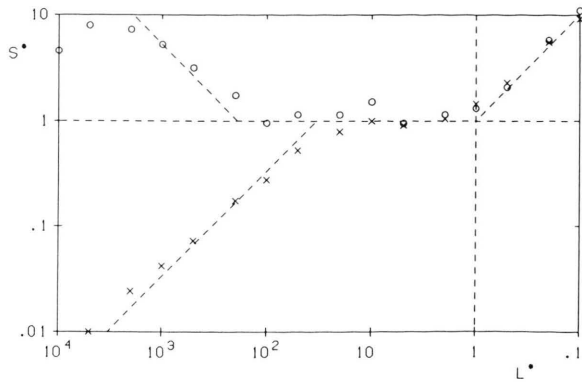


Fig. 1. Monte Carlo simulation of loss rates in a tokamak without and with ripple. Shown are normalized (S^*) loss rates (S) versus normalized (L^*) mean free paths (A): $S^* = S/S_p$, $S_p = 3.6v/(Q_0^2 \iota \pi R_0)$, v particle velocity, $Q_0 = a/\rho$, a plasma radius, ρ gyro-radius, ι rotational transform, R_0 toroidal radius; $L^* = A/L_c$, $L_c = \pi R_0/\iota$ half the connection length. Also shown are normalized loss rates $S_{PS}^* = 1/L^*$ (Pfirsch-Schlüter rate), $S_B^* = A^{1.5}/L^*$ (banana loss), $S_R^* = 1.65 \delta_e^{1.5} L^*$ (ripple loss), δ_e effective ripple. $Q_0 = 10^3$, $R_0/a = 10$, $\iota = 0.5$, $\delta_e = 0.02$.

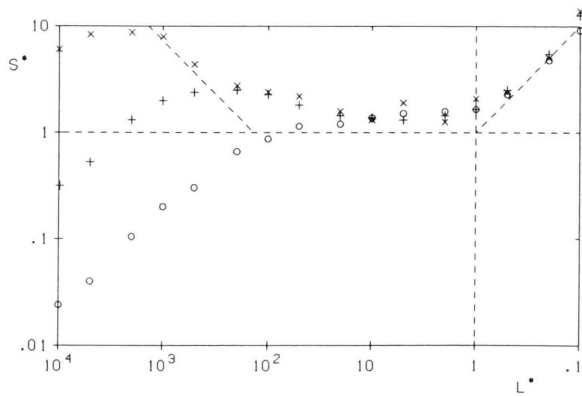


Fig. 2. Normalized loss rates in an $l=2$ stellarator for $Q_0 = 50$ (\circ), 200 ($+$), and 1000 (\times). $R_0/a = 8.6$, $N = 5$, $\iota = 0.5$, $\delta_e \approx 0.02$ (at aspect ratio $A = 17$).

stellarators was constructed by noting that the boundary shape of the plasma completely determines its physical properties, by suitably parametrizing this shape, and by searching the configurational space with 3D equilibrium codes supplemented by the stability assessment.

Figure 4 shows the geometrical characteristics of a Helias plasma column. Figure 5 shows zero- β flux surfaces of a specific Helias case [6, 7], and Fig. 6 the corresponding $\langle \beta \rangle = 0.15$ result.

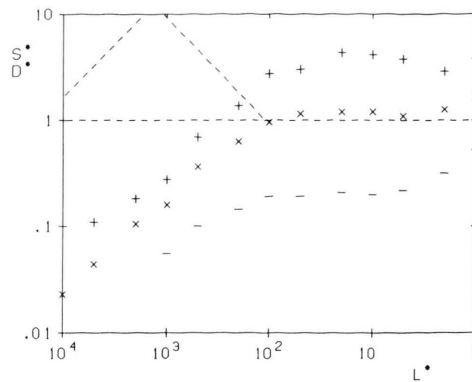


Fig. 3. Normalized loss rates S^* (\circ) in an $l=2$ stellarator with $e\Phi_0/E = \pm 2$; $Q_0 = 500$, $R_0/a = 8.6$, $N = 5$, $\iota = 0.5$, $\delta_e \approx 0.02$ (at $A = 17$). In addition, the corresponding normalized transport coefficient D^* (\times) (which does not depend on the sign of the electric field) is shown; $Q_0 = 500$, $A = 12.5$, $N = 5$, $\iota = 0.5$, $\delta_e \approx 0.03$. The L^* and L^{*-1} regimes are also indicated.

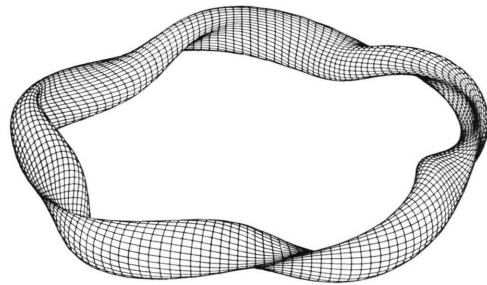


Fig. 4. Plasma boundary of a Helias configuration viewed obliquely from above.

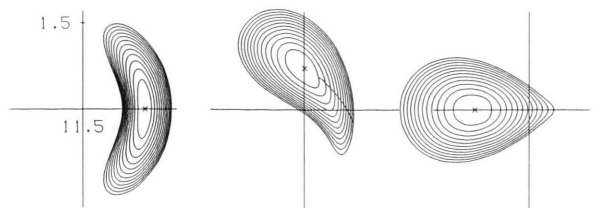
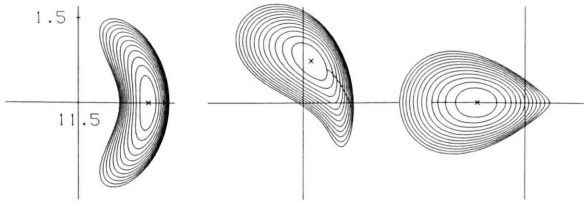


Fig. 5. Flux surfaces of a Helias equilibrium with $A = 11.5$, $N = 5$, $R_{0,1} = 0.8$, $Z_{0,1} = 0.4$, $A_{1,0} = 0.1$, $A_0 = 0.07$, $A_{1,-1} = 0.39$, $A_{2,0} = 0.05$, $A_{2,-1} = 0.24$, $A_{2,-2} = 0.07$, $\langle \beta \rangle = 0$, as obtained with the VMEC [8] code.

To consider ballooning modes in general 3D equilibria [9], Boozer's coordinates [10] were constructed [11] from the equilibrium information. These poloidal (θ) and toroidal (φ) coordinates are related to the usual poloidal (u) and toroidal

Fig. 6. Same as Fig. 5 but with $\langle \beta \rangle = 0.15$.

($v \propto$ toroidal angle of the cylinder coordinate system) coordinates of 3 D flux-variable codes by the following set of equations:

$$\vec{B} = \nabla s \times \nabla \psi = \nabla \chi + \beta \nabla s,$$

$$\psi = -F_T' u + F_P' v + \lambda,$$

$$\chi = J u + I v + \tilde{\chi},$$

$$F_T = F_T(s), \quad F_P = F_P(s), \quad J = J(s), \quad I = I(s), \quad ' = \frac{d}{ds};$$

$$\theta = u + \tilde{\theta},$$

$$\varphi = v + \tilde{\varphi},$$

$$\tilde{\theta} = (F_P' \tilde{\chi} - I \lambda) / (F_T' I + F_P' J),$$

$$\tilde{\varphi} = (F_T' \tilde{\chi} + J \lambda) / (F_T' I + F_P' J),$$

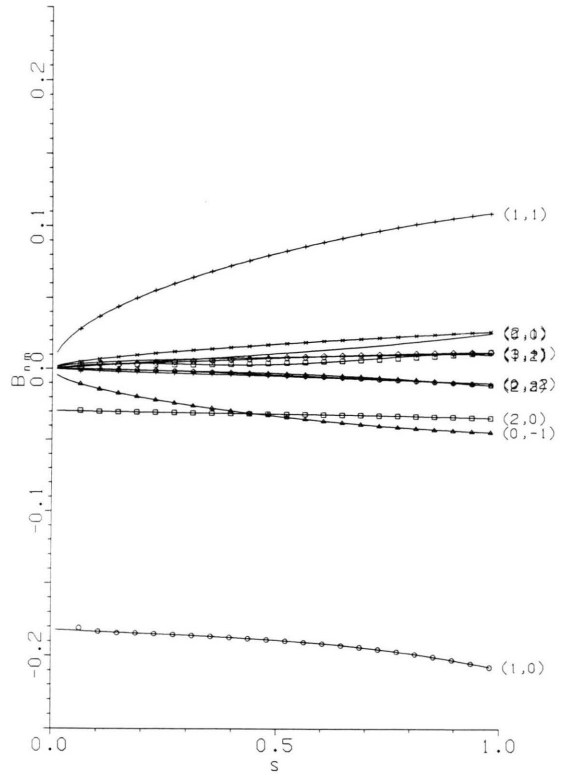
$$\psi = -F_T' \theta + F_P' \varphi,$$

$$\chi = J \theta + I \varphi$$

which shows that, in these coordinates, both the field lines and their orthogonals are straight. Because of

$$B^2 = (\nabla s \times \nabla \psi) \cdot \nabla \chi = -\frac{1}{\sqrt{g}} (F_T' I + F_P' J)$$

it is easy to see [10] that the guiding centre drift equations only require a knowledge of $\text{mod } \vec{B} = B(s, \theta, \varphi)$ in these coordinates. Figures 7 and 8 contain this information in Fourier-analyzed form. The three components $B_{0,0}$ (main magnetic field containing the deepening of the magnetic well at finite β), $B_{1,1}$ (representing the helical curvature), and $B_{1,0}$ (representing the mirror field with its minimum in the nearly straight part of the plasma column) are the dominant components. Furthermore, all components except $B_{0,0}$ depend only weakly on β . This is a consequence of the weak change in geometry of the flux-surfaces as β is increased (Figs. 5 and 6), which itself is of course due to the small Pfirsch-Schlüter current density.

Fig. 7. Fourier components B_{nm} ($n \cong \varphi$, toroidal index, $m = \theta$, poloidal index) of the magnetic field strength B as functions of the flux label s for the zero- β case shown in Fig. 5.

3. Results of Monte Carlo Simulations

The calculation of a global confinement time is done by the following Monte Carlo procedure, which relies on obtaining an asymptotic stationary distribution [1]. Only scattering by a uniform background plasma is simulated, so that a constant value of mean free path for pitch angle scattering is assumed throughout the plasma volume. A particle leaving this volume is replaced by another particle with the same initial energy E in such a way that a particle (with index n_i) of the remaining test distribution is doubled and from then on treated as statistically independent. The particle to be doubled is selected according to a cyclic procedure

$$n_i = (n_{i-1} + k_{\text{cyc}}) \text{ modulo } 64$$

for a test distribution which comprises 64 particles. Here, n_{i-1} is the index of the particle which was

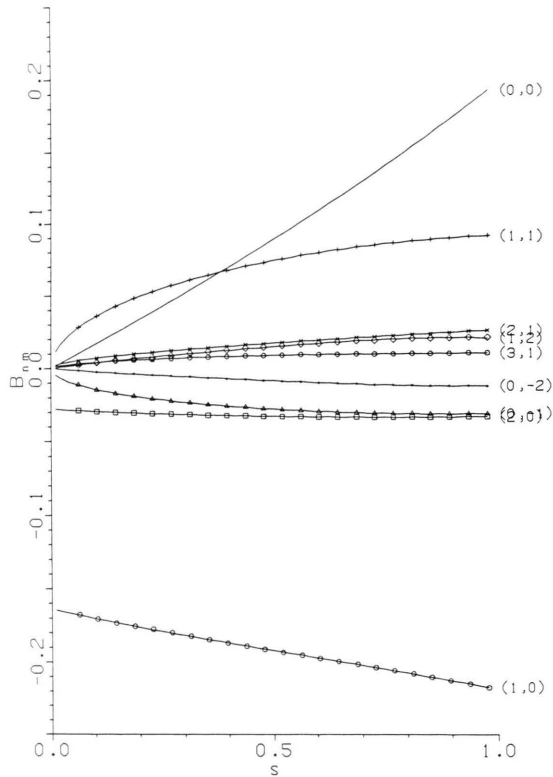


Fig. 8. Same as Fig. 7 but with $\langle \beta \rangle = 0.15$.

doubled last. The cycle number k_{cyc} has to be relatively prime to the number of particles. By this procedure a stationary solution is asymptotically obtained and corresponds to the slowest decay mode of the drift kinetic equation without source term. The decay rate of this mode corresponds to the Monte Carlo replenishment rate and is equivalent to the loss rate. Asymptotically in time, the procedure described is equivalent to a particle source function proportional to the distribution function.

For a tokamak without electric field, the following plateau loss rate is found:

$$S_P = \frac{3.6 v}{Q_e^2 l \pi R_0}. \quad (3.1)$$

It is useful to keep the following sets of numbers in mind:

$$\begin{aligned} E (\text{deuterons}) &= 10 \text{ keV}, \quad a = 1 \text{ m}, \quad R = 10 \text{ m}, \\ B &= 5 \text{ T}, \quad l = 1/2 \text{ yields } S_P \approx 4 \text{ s}^{-1}. \\ E (\text{protons}) &= 3 \text{ keV}, \quad a = 0.5 \text{ m}, \quad R = 5 \text{ m}, \\ B &= 3 \text{ T}, \quad l = 1/2 \text{ yields } S_P \approx 9 \text{ s}^{-1}. \end{aligned}$$

Normalized loss rates are introduced by

$$S^* = S/S_P.$$

For the Pfirsch-Schlüter, banana, and, in the case of a rippled tokamak, ripple regimes, the following results are found:

$$S_{PS}^* = 1/L^*; \quad S_B^* \approx A^{3/2}/L^*; \quad S_R^* = 1.65 \delta_e^{3/2} L^*.$$

Figures 1 and 2 show these results and, in the Imfp regime, the relation

$$S_v \approx v/A = v.$$

With an electric field, the pitch angle collision operator has to be supplemented by an energy relaxation operator which keeps the particle kinetic energy approximately constant. For, if the total energy were conserved in the collisions, the electrostatic confinement properties would dominate the behaviour of the test particle distribution function. Here, the kinetic energy is relaxed with the same time constant as governs the pitch angle scattering:

$$\begin{aligned} x_{\text{new}} &= x_{\text{old}} (1 - \Delta) \pm \sqrt{(1 - x_{\text{old}}^2) \Delta}, \quad x = v_{\parallel}/v, \\ E_{\text{new}} &= E_{\text{old}} + (E - E_{\text{old}}) \Delta, \end{aligned} \quad (3.2)$$

where $\Delta = \Delta t/\tau_{90^\circ}$ (Δt time step) and x_{new} , x_{old} are the values of the pitch angle and E_{new} , E_{old} the values of the kinetic energy, E being the initial energy of the test particle. This procedure results in thermodynamic equilibrium of the test particle distribution f in the applied potential Φ , i.e. $f = f_0 \exp(-e\Phi/E)$, where f_0 is the distribution function without applied electric potential.

From the procedure described above it is plausible that a guiding centre particle, since its collisionless motion is governed by energy conservation

$$H = \frac{m}{2} v_{\parallel}^2 + \mu B + e \Phi,$$

should be better confined in a true minimum- B configuration. Figure 9 shows the radial (s) dependence of the minimum and maximum of B for the $\beta = 0$ and the $\langle \beta \rangle = 0.15$ cases considered in Section 2. The finite- β Helias case is indeed characterized by a true minimum of B (see Fig. 9), while in classical $l = 2$ stellarators this situation cannot be realized because the equilibrium β limit is too small to compensate the toroidal effect.

Figure 10 shows the results of the loss rate calculation for the $\beta = 0$ and $\langle \beta \rangle = 0.15$ cases. In the

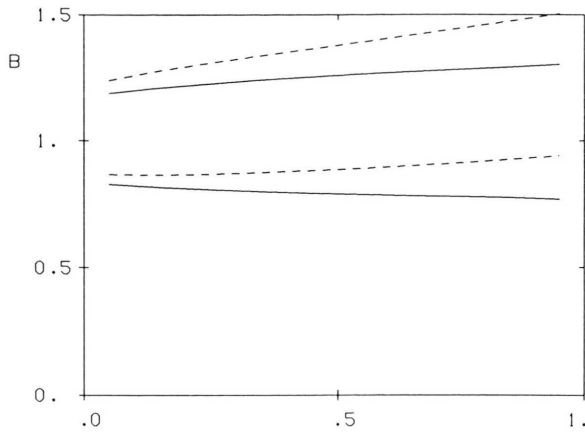


Fig. 9. Minimum and maximum of B as functions of the radial label s for the $\beta=0$ (—) and the $\langle\beta\rangle=0.15$ (---) Helias cases.

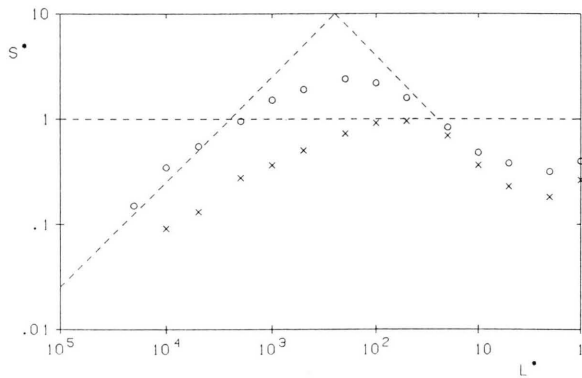


Fig. 10. Normalized loss rates S^* in the $\beta=0$ (○) and in the $\langle\beta\rangle=0.15$ (×) Helias cases as functions of normalized mean free paths.

latter case the confinement is improved by a factor of about 4 in the Imfp regime.

Figures 11 and 12 show the corresponding results for an applied electric potential characterized by $e\Phi_0/E=0.5$. For the $\beta=0$ case confinement times are already improved by one order of magnitude with this relatively small potential, irrespective of its sign. For the finite- β case the “repelling” potential has only a small beneficial effect. This result is of course to be expected and can also be stated in the form that, for the “repelling” potential, an increase in β may decrease confinement times. Correspondingly, the “attracting” potential is very

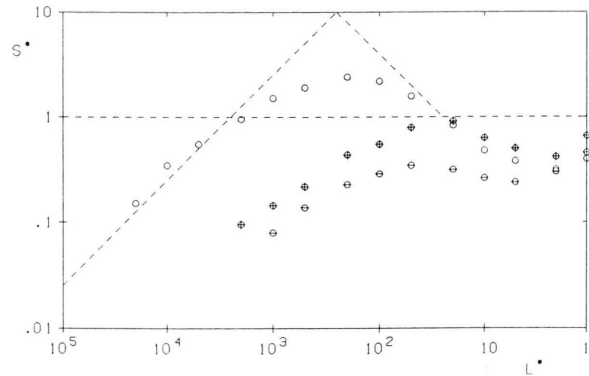


Fig. 11. Normalized loss rates S^* in the $\beta=0$ case with an electrostatically attracting (−) and a repelling (+) potential given by $e\Phi_0/E=\pm 0.5$.

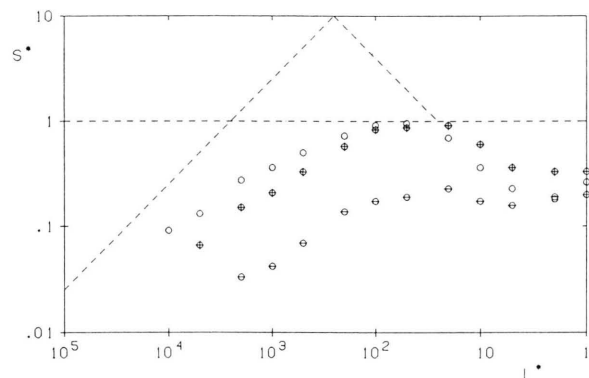


Fig. 12. Same as Fig. 11, but with $\langle\beta\rangle=0.15$.

beneficial in the finite- β case and leads to very good neoclassical ion confinement comparable to the tokamak banana regime.

4. Conclusion

For the Helias class of stellarators it has been demonstrated that at finite β a true minimum- B situation can be established which, in the case of an attracting potential, is beneficial for global ion confinement to the extent that basic fusion reactor experiments might be satisfied.

- [1] W. Lotz, J. Nührenberg, and A. Schlüter, *J. Comp. Phys.* (1987), in print.
- [2] R. H. Fowler, J. A. Rome, and J. F. Lyon, *Phys. Fluids* **28**, 338 (1985).
- [3] H. E. Mynick, *Phys. Fluids* **28**, 1139 (1985).
- [4] J. Nührenberg and R. Zille, *Phys. Letters* **114 A**, 129 (1986).
- [5] S. P. Hirshman, J. Nührenberg, and R. Zille, 8th Europhysics Conf. on Comp. Physics, Eibsee 1986, ECA Vol. **10 D**, 157–160 (1986).
- [6] W. Dommaschk, F. Herrnegger, W. Lotz, P. Merkel, J. Nührenberg, A. Schlüter, U. Schwenn, and R. Zille, 11th Int. Conf. on Plasma Physics and Contr. Nuclear Fusion Res., Kyoto 1986, IAEA-CN-47/D-I-3.
- [7] W. Dommaschk, F. Herrnegger, W. Lotz, P. Merkel, J. Nührenberg, A. Schlüter, U. Schwenn, and R. Zille, Proc. Int. Stellarator/Heliotron Workshop (IAEA Technical Committee Meeting), Kyoto 1986, Plasma Physics Lab., Kyoto University, Rep. PPLK-6, Vol. **II**, 336 (1987).
- [8] S. P. Hirshman, P. Merkel, and W. I. van Rij, *Comp. Phys. Comm.* **43**, 143 (1986).
- [9] D. Correa-Restrepo, *Z. Naturforsch.* **33 a**, 789 (1978).
- [10] A. Boozer, *Phys. Fluids* **23**, 904 (1980).
- [11] J. Nührenberg and R. Zille, Proc. 12th Eur. Conf. on Contr. Fus. and Plasma Phys., Budapest 1985, ECA Vol. **9 F**, I, 445 (1985).